GUIA 2

1) a) $\operatorname{tr}(ST) = (ST)_{ii}$

$$= \underline{S_{ij} T_{ji}} = T_{ji} S_{ij} = (TS)_{jj}$$

$$= \operatorname{tr}(TS)$$

$$= S^T_{ji} T^T_{ij} = (S^T T^T)_{jj} = \operatorname{tr}(S^T T^T)$$

4) $(T^T)^T = T$

$$\left[(T^T)^T\right]_{ij} = (T^T)_{ji} = T_{ij}$$

$$= [T]_{ij}$$

$$\Rightarrow (T^T)^T = T$$

4c) $\left[(ST)^T\right]_{ij} = (ST)_{ji}$

$= S_{jk} T_{ki} = T^T_{ik} S^T_{kj} = (T^T S^T)_{ij}$

$\Rightarrow (ST)^T = T^T S^T$

6) $$I^{\pm}_{ijkl} = \frac{1}{2}\left(\delta_{ik}\delta_{jl} \pm \delta_{il}\delta_{jk}\right)$$

Para $T \in CT(2)$:

$$I^{\pm}_{ijkl} T_{kl} = \frac{1}{2}\left(\delta_{ik}\delta_{jl} \pm \delta_{il}\delta_{jk}\right) T_{kl}$$

$$= \frac{1}{2}\left(T_{il}\delta_{jl} \pm T_{ki}\delta_{jk}\right) = \frac{1}{2}\left(T_{ij} \pm T_{ji}\right)$$

Para $S \in CT(2)$ simétrico:

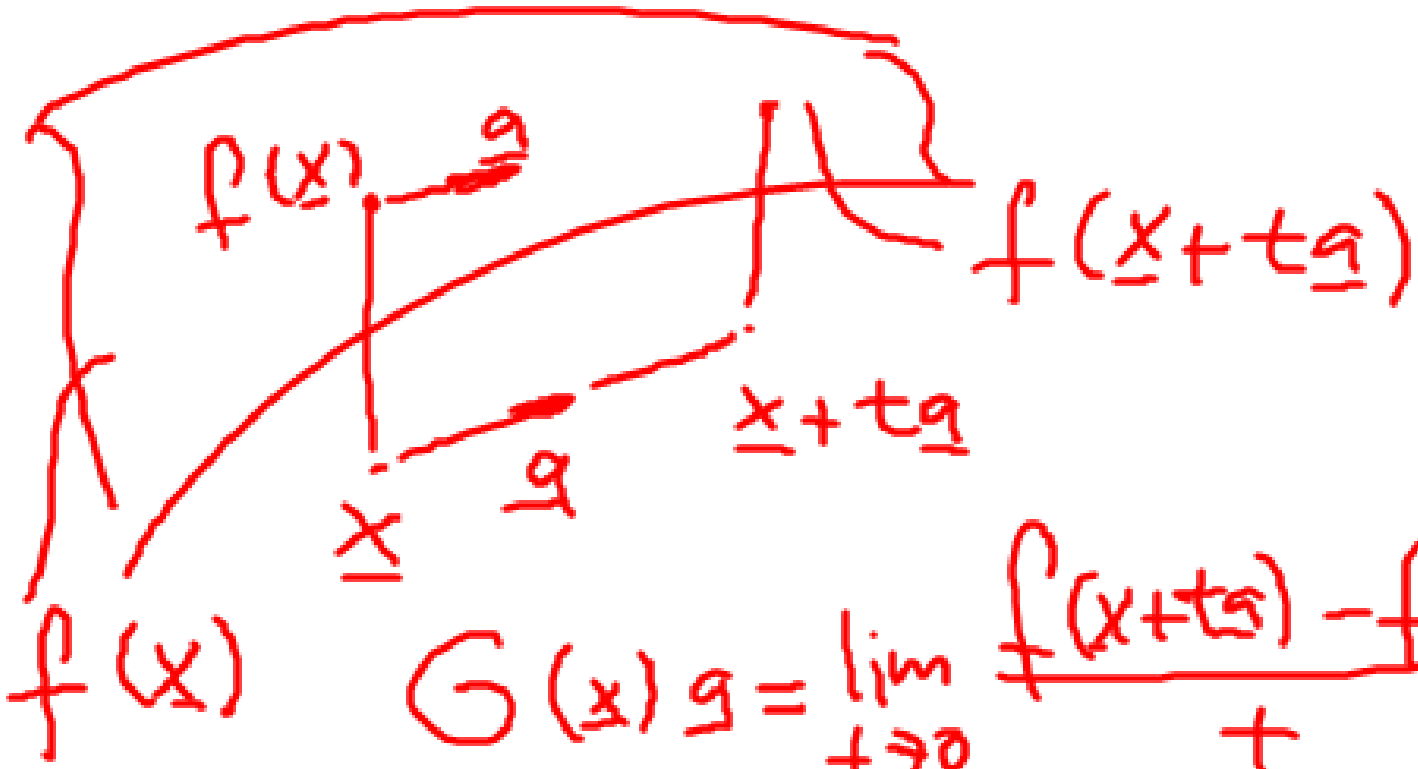
$$I^{+}_{ijkl} S_{kl} = S_{ij}$$

I^{+}_{ijkl} : tensor identidad de orden 4 para $CT(2)$ simétricos

Representación espectral

$C \in CT(2)$ simétrico y positivo def.

$$\underline{\underline{C}} = C_{ij}\, e_i \otimes e_j$$
$$= \sum_i \lambda_i\, v^{(i)} \otimes v^{(i)}$$



$$G(\underline{x})\,\underline{a} = \lim_{t\to 0} \frac{f(\underline{x}+t\underline{a}) - f(\underline{x})}{t}$$

Si $f \in CT(n)$, $Ga \in CT(n)$

$$\underbrace{G_{ij\cdots m}}_{n+1 \text{ indices}} a_m = (Ga)_{\underbrace{ij\cdots}_{n \text{ indices}}}$$

$$(\nabla \otimes v)_{ij}\, a_j$$

$$(\nabla \otimes v)_{ij} = \begin{cases} \dfrac{\partial v_i}{\partial x_j} \; ? \\[2ex] \dfrac{\partial v_j}{\partial x_i} \; ? \end{cases}$$

$$(\nabla \otimes v)\, a = \lim_{t \to 0} \frac{v(x + t a) - v(x)}{t}$$

$$\text{Si } a = e_1 \Rightarrow (\nabla \otimes v)_{ij}\, \delta_{1j} = \lim_{t \to 0} \frac{v_i(x + t e_1) - v_i(x)}{t}$$

$$(\nabla \otimes v)_{i1} = \frac{\partial v_i}{\partial x_1}$$

$$\underbrace{(\mathrm{rot}\,\underline{v})}_{\underline{w}} \cdot \underline{a} = \mathrm{div}\,(\underline{v} \times \underline{a})$$

$$\mathrm{div}\,(\underline{v} \times \underline{a}) = \frac{\partial}{\partial x_i} (v \times a)_i = \frac{\partial}{\partial x_i} (\varepsilon_{ijk} v_j a_k)$$

$$= \varepsilon_{ijk} \frac{\partial v_j}{\partial x_i} a_k$$

$$(\mathrm{rot}\,\underline{v})_k \cancel{a_k} = \varepsilon_{ijk} \frac{\partial v_j}{\partial x_i} \cancel{a_k}$$

$$(\mathrm{rot}\,\underline{v})_k = \varepsilon_{kij} \frac{\partial v_j}{\partial x_i} = \varepsilon_{kij} \left[\mathrm{grad}\,\underline{v}\right]_{ji}$$

Si $T \in C^1(\Omega)$

$$(\text{grad}\, T)_{ijk} = \frac{\partial T_{ij}}{\partial x_k}$$

$$\frac{\partial T_{ij}}{\partial x_j} = \left[\nabla \cdot T^T\right]_i$$

$$\underline{(\nabla \cdot T) \cdot a} = \nabla \cdot (Ta)$$

$$\nabla \cdot (Ta) = (Ta)_{i,i} = (T_{ij} a_j)_i = \frac{\partial T_{ij}}{\partial x_i} a_j$$

$$(\nabla \cdot T)_j a_j = \frac{\partial T_{ij}}{\partial x_i} a_j$$

T. de Gauss

$$\int_D \frac{\partial T_{ik}}{\partial x_j}\, dV = \int_S T_{ik} n_j \, dS$$

$$\int_D (\nabla \cdot T) \cdot w \, dV = \int_S \overbrace{(Tn)}^{t} \cdot w \, dS - \int_D T : \nabla^s w \, dV$$

$$\nabla^s w = \frac{1}{2}\left[\nabla w + (\nabla w)^T\right]$$

$$(\nabla \cdot T)_i w_i = \left(\frac{\partial}{\partial x_j} T_{ji}\right) w_i = \underbrace{\frac{\partial}{\partial x_j}(T_{ji} w_i)} - T_{ji}\frac{\partial w_i}{\partial x_j}$$

$$\int_D (\nabla \cdot T) \cdot w \, dV = \int_D \frac{\partial}{\partial x_j}\left(T_{ji} w_i\right) dV - \int_D T_{ji} \frac{\partial w_i}{\partial x_j} dV$$

$$= \int_S T_{ji} w_i n_j \, dS - \int_D T_{ji} \frac{\partial w_i}{\partial x_j} dV$$

$$= \int_S \left(T_{ij} n_j\right) w_i \, dS - \int_D T_{ji} \frac{\partial w_i}{\partial x_j} dV$$

$$= \int_S \underline{\underline{(Tn) \cdot w}} \, dS - \int_D T_{ji} \frac{\partial w_i}{\partial x_j} dV$$

$$T_{ji}\frac{\partial w_i}{\partial x_j} = T_{ij}\frac{\partial w_j}{\partial x_i} = \frac{1}{2}\left(\overbrace{T_{ji}\frac{\partial w_i}{\partial x_j} + T_{ij}\frac{\partial w_j}{\partial x_i}}^{=}\right)$$

$$= T_{ij}\underbrace{\frac{1}{2}\left(\frac{\partial w_i}{\partial x_j} + \frac{\partial w_j}{\partial x_i}\right)}_{(\nabla \otimes^s w)_{ij}} = T : \nabla \otimes^s w$$

$$\operatorname{div}\underbrace{(\operatorname{rot} v)}_{w \in C^1(\Omega)} = \frac{\partial}{\partial x_i}\Big(\overbrace{\varepsilon_{ijk}\frac{\partial v_k}{\partial x_j}}^{w_i}\Big) = \Big(\varepsilon_{ijk}\frac{\partial^2 v_k}{\partial x_i \partial x_j}\Big)$$

$$= \varepsilon_{jik}\frac{\partial^2 v_k}{\partial x_j \partial x_i} = -\Big(\varepsilon_{ijk}\frac{\partial^2 v_k}{\partial x_i \partial x_j}\Big) = 0$$

par simetria

$$\operatorname{div}(\operatorname{rot}\underline{v}) = \left(\varepsilon_{ijk}\frac{\partial v_k}{\partial x_j}\right)_{,i} = \varepsilon_{ijk}\frac{\partial^2 v_k}{\partial x_j \partial x_i}$$

$$= \frac{1}{2}\left(\varepsilon_{ijk}\frac{\partial^2 v_k}{\partial x_j \partial x_i} + \varepsilon_{jik}\frac{\partial^2 v_k}{\partial x_i \partial x_j}\right)$$

$$= \frac{1}{2}\left(\varepsilon_{ijk} + \varepsilon_{jik}\right)\frac{\partial^2 v_k}{\partial x_i \partial x_j}$$

$$= \frac{1}{2}\left(\varepsilon_{ijk} - \varepsilon_{ijk}\right)\frac{\partial^2 v_k}{\partial x_i \partial x_j} = 0$$